Instabilities in nuclei

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Abstract. The evolution of dynamical perturbations is examined in nuclear multifragmentation in the frame of Vlasov equation. Both plane wave and bubble type of perturbations are investigated in the presence of surface (Yukawa) forces. An energy condition is given for the allowed type of instabilities and the time scale of the exponential growth of the instabilities is calculated. The results are compared to the mechanical spinodal region predictions.

1. Introduction

Let us consider a spherically expanding nuclear system in the metastable nuclear fluid phase when it reaches the freeze-out at time τ_{fr}. Although at the freeze-out the fermionic degrees of freedom are frozen-out, and internucleon collisions cease, softer long range nuclear interactions are still effective, and represented by a nuclear mean field potential, $U(\vec{r})$.

We will assume that the system, both before and after freeze-out undergoes a spherical, scaling expansion. Such an expansion can be represented by a four-velocity field, $u_\mu = x^\mu/\tau$, where $\tau = \sqrt{t^2 - x^2 - y^2 - z^2}$, for the internal regions of our expanding system (but not for the external surface). This flow pattern is invariant under Lorentz transformation, i.e., the points of the interior of our expanding system are physically identical and indistinguishable from one another. Consequently in the interior, in the Local Rest (LR) frame all thermodynamical and fluid-dynamical quantities are equal, and from the point of view of instabilities all internal points are equivalent. We will exploit these symmetry features although in the calculations we will use a non-relativistic approximation.

Let us assume that the nucleon phase space distribution before, and at the freeze-out, τ_{fr}, is a Fermi distribution:

$$f_0(\tau_{fr}, \vec{r}, \vec{p}) = C \left\{ 1 + \exp\left[\frac{[\vec{p} - m\vec{r}/\tau_{fr}]^2}{2mT_{fr}} - \frac{\mu_{fr}}{T_{fr}} \right] \right\}^{-1} \tag{1}$$

where this form assumes a correlation between the momentum and radial distribution arising from radial expansion, $C = g/(2\pi\hbar)^3$ is the normalization, and g is the degeneracy of nucleons, so that the proper (LR) density is $n_0(\tau_{fr}, \vec{r}) = \int d^3p \, f_0(\tau_{fr}, \vec{r}, \vec{p})$. We assume that in the interior of the collision zone the freeze-out density is constant: $n_0(\tau_{fr}, \vec{r}) = n_{fr}$. In the center of the collision zone this is an ideal Fermi distribution, while at finite radii, $|\vec{r}|$, the distribution is boosted (using non-relativistic, Galilei transformation), with a radially directed and radially linearly increasing flow velocity of $\vec{v} = \vec{r}/\tau$. We can also introduce the LR momentum: $\vec{P}(\tau, \vec{r}) = \vec{p} - m\vec{r}/\tau$.

Furthermore, let us assume that after the freeze-out for $\tau > \tau_{fr}$, the system expands homogeneously according to the collisionless Vlasov equation. The distribution function, f, is the solution of the Vlasov equation with a mean-field potential, $U(\vec{r})$,

$$\frac{\partial f}{\partial \tau} + \frac{\vec{p}}{m}\frac{\partial f}{\partial \vec{r}} - \frac{\partial U}{\partial \vec{r}}\frac{\partial f}{\partial \vec{p}} = 0. \tag{2}$$

In the special case of a homogeneous system, where the last term vanishes, for such a free coasting expansion in the local rest frame is just obtained by replacing $\vec{p}$ by $\tau\vec{p}/\tau_{fr}$ in Eq. (1) [1, 2]:

$$f_0(\tau, \vec{r}, \vec{p}) = C \left\{ 1 + \exp\left[\frac{[\vec{p} - m\vec{r}/\tau]^2}{2\, m\, T_{eff}} - \frac{\mu_{eff}}{T_{eff}} \right] \right\}^{-1} \tag{3}$$

where $T_{eff} = T_{fr}(\tau_{fr}/\tau)^2$ and $\mu_{eff} = \mu_{fr}(\tau_{fr}/\tau)^2$. The condition, that the ratio of the chemical potential and the temperature is constant during the expansion, in a usual thermodynamical system, corresponds to an adiabatic process. The density of the system changes with time in this inertial expansion as $n_0(\tau) = n_{fr}(\tau_{fr}/\tau)^3$. This solution is valid starting from the freeze-out, τ_{fr}, and until inhomogeneities will spontaneously develop in the system at some threshold time, τ_{th}. Before this time small perturbations will smooth out due to the mean field potential, while after this threshold time the density dependent mean field will enhance fluctuations. So after this threshold time the density will not be homogeneous any more.

Note that this post freeze-out distribution, f, is not a thermal equilibrium distribution function, and the effective parameters, T_{eff} and μ_{eff} are just carrying the memory of the last equilibrium thermal parameters, T_{fr} and μ_{fr}, but these are not the usual thermodynamical parameters. This can be easily seen if the expansion is not spherically symmetric [1].

If we would have a thermal expansion following τ_{fr}, the Equation of State (EOS) would determine the time dependence of the physical temperature in an adiabatic expansion. Generally this would not coincide with $T_{eff} = T_{fr}(\tau_{fr}/\tau)^2$, only in the case of a large system, where the flow dominates the energy. For example if the EOS is that of an ideal Stefan-Boltzmann gas, ($\partial p/\partial e = c_0^2$, $c_0^2 = 1/3$ and $e = cT^4$), then $T(\tau) = T_{fr}(\tau/\tau_{fr})^{-3c_0^2}$, which differs from T_{eff}.

We study the stability of the system and the occurrence of instabilities arising from the mean field. Such an instability may lead to a rapid multifragmentation of our system.

2. Instabilities

Let us consider a small perturbation in the expanding system. The amplitude of this perturbation may grow, decrease or oscillate depending on the conditions. In the presence of such a perturbation the phase space distribution is:

$$f(\tau, \vec{r}, \vec{p}) = f_0 + f_1(\tau, \vec{r}, \vec{p}),$$

with the normalization $n(\tau, \vec{r}) = \int d^3p \,(f_0 + f_1)$ and $n_1(\tau, \vec{r}) = \int d^3p \, f_1$, where the unperturbed density n_0 is homogeneous, $\vec{\nabla} n_0 = 0$, and f_1 should be a local spherical perturbation which is a solution of the Vlasov equation

$$\frac{\partial f_0}{\partial \tau} + \frac{\partial f_1}{\partial \tau} + \frac{\vec{p}}{m}\left[\frac{\partial f_0}{\partial \vec{r}} + \frac{\partial f_1}{\partial \vec{r}}\right] - \vec{\nabla}U\left[\frac{\partial f_0}{\partial \vec{p}} + \frac{\partial f_1}{\partial \vec{p}}\right] = 0. \qquad (4)$$

The solution of the Vlasov equation is treated in details in Ref. [3]. Here we separate the Vlasov equation into two equations, one for f_0 and one for f_1. Separating non-vanishing dominant zeroth order terms we get the equation

$$\frac{\partial f_0}{\partial \tau} + \frac{\vec{p}}{m}\frac{\partial f_0}{\partial \vec{r}} = 0,$$

which is satisfied by f_0 as given in Eq. (3). The first order terms yield the linearized equation

$$\frac{\partial f_1}{\partial \tau} + \frac{\vec{p}}{m} \frac{\partial f_1}{\partial \vec{r}} - \vec{\nabla} U \frac{\partial f_0}{\partial \vec{p}} = 0.$$

We intend to find perturbations which grow, leading to instabilities of the system. Some modes of growing perturbations may arise in thermal surrounding, and their rate is determined by thermal and viscous damping [4, 5, 6, 7, 8]. These are usually slower, and so other faster processes may come into play also. Here we intend to study non-thermal, growing perturbations, which may occur after the thermal freeze-out only, but they can be faster than the thermally damped processes [1, 9, 5, 6, 10, 11]. The growth rate of such perturbations is determined by the long range nuclear mean field potential, $U(\vec{r})$. Usually different non-thermal channels of instability open only when a given time is passed after the freeze-out at τ_{fr}, and we reach a threshold time, τ_{th}.

There are two characteristic time scales in the system: (i) the longer post freeze-out expansion between τ_{fr} and τ_{th}, and (ii) the rapid growth of instability which develops after τ_{th}. When studying the dynamics of rapidly growing instabilities we can usually neglect the much slower dynamics of the post freeze-out expansion.

Different configurations can and should be taken into account when studying instabilities in a quenched (supercooled) system.

2.1. Plane wave perturbation

The stability of the Vlasov equation against plane wave perturbations was examined in detail by different groups recently [12, 13]. If we expand the perturbation, f_1, as

$$f_1(\vec{r}, \vec{p}, t) = \sum_k f_k(p, t) \frac{1}{\sqrt{\Omega}} e^{i\vec{k}\vec{r}}, \tag{5}$$

and search for the solution of $f_k(p, t)$ in the form

$$f_k(p, t) = f_{k,\omega}(p) e^{i\omega t} \quad ,$$

according to Ref. [12] we get the dispersion relation $\omega(k)$

$$1 = \frac{\partial U(k)}{\partial n} \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{(\vec{k}\vec{p})^2}{(\vec{k}\vec{p})^- m^2\omega^2} \frac{\partial \tilde{f}}{\partial \epsilon} \quad , \tag{6}$$

where $\tilde{f}$ is the static uniform solution, and $U(k)$ denotes the Fourier component of the effective field $U(\vec{r})$. The mode corresponding to wavenumber k will be unstable, when the $\omega(k)$ frequency becomes imaginary. It was also shown in Ref. [12], that for zero temperature the condition of instability can be written as

$$\frac{2}{3}\epsilon_F + n\frac{\partial U(k)}{\partial n} < 0 \quad , \tag{7}$$

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with ϵ_F Fermi kinetic energy. The expression goes over for $k \rightarrow 0$ into the condition of *mechanical instability* (i.e., the compressibility becomes negative)

$$\left(\frac{\partial p}{\partial n}\right)_{T=0} = \frac{\partial}{\partial n}\left(n^2\frac{\partial \epsilon}{\partial n}\right) = \frac{2}{3}\epsilon_F + n\frac{\partial U(n)}{\partial n} < 0 \quad,$$

where the potential $U(n)$ depends only on the homogeneous density.

In Ref. [13] the instability condition was examined for a one dimensionally expanding ground state system. The acting force was a Skyrme force with Yukawa surface term. The resulting instability condition reads as Eq. (7), where

$$\frac{\partial U(k)}{\partial n} = -2\beta + \gamma(\sigma+1)(\sigma+2)n_0^\sigma - \frac{4\pi V_0}{\mu^2 + k^2} \quad,$$

and β, γ, σ, V_0 and μ are the Skyrme and Yukawa force parameters (see Eq. (11)).

In case of a uniformly expanding system the same condition of instability for a plane wave perturbation can also be obtained in a fashion similar to the spherical case discussed later, see Appendix A. This approach leads to the following condition

$$1 = -\frac{2\pi\tau_{fr}mC}{\tau}\sqrt{2mT_{fr}}\,\frac{\partial U(k)}{\partial n}\int\limits_0^\infty \frac{dy\,\sqrt{y}}{(y+s)[1+e^{y-\mu_{fr}/T_{fr}}]} \quad, \tag{8}$$

where $s = m\kappa^2\tau^4/(2k^2 T_{fr}\tau_{fr}^4)$, $\kappa = i\omega$ and $\partial U(k)/\partial n$ is the same expression as in Ref. [13].

2.2. Spherical drop/bubble perturbations

In the following we want to study more realistic perturbations instead of plane waves. We consider local spherical bubbles, since we believe these are the first instabilities which start to grow [14]. In general spherical perturbations minimize the surface and surface energy, so these can be formed the earliest. (On the other hand plane wave perturbations may grow more rapidly at later stages with stronger driving forces due to the increased surface.)

Consider a spherical drop with a surface density profile exponentially decreasing characterized by the parameter k, a central density, n_c, and radius $R(\tau)$

$$f_1 = f_s\begin{cases} n_c\exp\left[-k\frac{\tau_{fr}}{\tau}(r-R(\tau))\right] & r \geq R(\tau) \\ \\ n_c & r < R(\tau) \end{cases} \quad, \tag{9}$$

where $R(\tau) = R^*\frac{\tau}{\tau_{th}}e^{\kappa(\tau-\tau_{th})}$ is the τ dependence of the radius after the threshold time, τ_{th} when the droplet becomes bigger than the critical radius and it will be able to grow. We are interested in the initial growth rate of the radius only, so

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that in $R(\tau)$ the exponential term can be expanded into a power series for small $\tau - \tau_{th}$, i.e.,

$$R(\tau) \approx R^* \frac{\tau}{\tau_{th}} [1 + \kappa(\tau - \tau_{th})].$$

Inserting this approximation into Eq. (9) we get for the exterior part $(r > R(\tau))$ of the profile:

$$f_1 = f_s n_c \exp\left(kR^*\frac{\tau_{fr}}{\tau_{th}}\right) \exp\left(-k\frac{\tau_{fr}}{\tau}r + kR^*\frac{\tau_{fr}}{\tau_{th}}\kappa(\tau - \tau_{th})\right).$$

Since we consider small perturbations only, where the linearization of Eq.(2) holds, this solution is valid only for a short time after the instability starts to grow. The dispersion relation will lead to a dynamical growth factor, κ, depending both on k and R.

(In the plane wave expansion of the perturbation studied in Ref. [12, 13] the opening of the channel of the instability was indicated when ω became imaginary. Thus perturbations preceding τ_{th} did not grow. Our approach is basically equivalent to their one presented here, however, we wanted to emphasize that the instability may grow only after τ_{th}.)

Since $f_s(\tau, \vec{P})$ has a characteristic time dependence on the slow scale (i) of the post freeze-out expansion, we assume that its time-derivatives are negligible compared to other time-derivatives of $f_1(\tau, \vec{r}, \vec{p})$ corresponding to rapid inequilibrium processes (ii) like $\exp[\kappa(\tau - \tau_{th})]$. Furthermore, we assume that f_s depends on the LR momentum, $\vec{P}$ only, i.e., it does not have any other dependence on $\vec{r}$ other than what is included in $\vec{P}$.

We search for such solutions of the Vlasov equation, $f_1(\tau, \vec{r}, \vec{p})$ and $f_s(\tau, \vec{P})$. We assume that such solutions can be obtained only some time, τ, after the freeze-out at τ_{fr}, i.e. at $\tau_{th} > \tau_{fr}$. Before this time thermal processes and thermal damping is dominant which generally lead to slower nucleation than post-freeze-out processes driven by the background fields.

We can calculate the critical droplet radius, R^*_{crit}. Droplets smaller than R^*_{crit} tend to disappear, while droplets larger than R^*_{crit} may start to grow. Thus we will study the growth rate of critical size droplets. The critical radius, R^*_{crit} is calculated in Appendix E.

The critical radius, R^*_{crit}, should be evaluated when the channel of instability opens at τ_{th}, and the critical droplet just starts to grow. In the 3-dimensional scaling expansion the critical radius scales with the overall scaling, which leads to a quasi-static critical radius of $R^*_{\text{crit}}\tau/\tau_{th}$.

The critical radius, R^*_{crit}, depends on the background nucleon density at the opening of the instability, $n_0(\tau_{th}) = n_{fr}(\tau_{fr}/\tau_{th})^3$, or consequently on τ_{th}, furthermore on the surface parameter, k, on the central density, n_c, and on the parameters of the interaction potential. The total energy of the system can be simultaneously minimized by varying k and n_c, and searching for an extremum as a function of R^*.

As we mentioned the time-scale of the expansion is assumed to be slow compared to the time-scale of the instability, so R^* can be considered as a time independent constant when studying the growth of instability.

The perturbation (9) satisfies the Vlasov equation both for the exterior and interior region, if an averaged Yukawa potential is used (see Appendix C).

The dispersion relation for such a spherical perturbation can be written as

$$1 = -\frac{2\pi\tau_{fr}mC}{\tau}\sqrt{2mT_{fr}}\,\frac{\partial U}{\partial n}\int_0^\infty \frac{dy\,\sqrt{y}}{(y-S)[1+e^{y-\mu_{fr}/T_{fr}}]} \tag{10}$$

(see Appendix B), where $S = m\kappa^2(R^*)^2\tau^4/(2T_{fr}\tau_{fr}^2\tau_{th}^2)$.

It is easy to see, that for $S = 0$ without the surface term this condition is equivalent to the isothermal mechanical instability even for $T \neq 0$ (Appendix D), that is the region of dynamical instability and of mechanical one coincide. Although the direct k-dependence drops out of the dispersion relation, but since R^* depends on k so the dispersion relation is still applicable. It is interesting to mention that both dispersion relations, (8,10), yield the same condition for evaluating the threshold time for the perturbation which is just on the boundary to be able to grow, i.e., $s = S = 0$.

The features of the potential and the Yukawa term in it are vital in determining the properties of the static, critical droplet or bubble. Physically we can consider two situations in the course of final multifragmentation.

Depending on the beam energy, after the initial compression we reach the most compressed and heated up state with a definite specific entropy. This stage is then followed by an expansion, which is adiabatic to a good approximation. If the final specific entropy is smaller than the critical entropy of the nuclear liquid-gas phase transition, the expansion will lead to a stretched (or quenched) liquid state, with density n_0 below the normal nuclear density, n_N. The instabilities will lead then to bubble formation with an interior nuclear gas phase density, $n_1 + n_0 \approx 0.1 - 0.4n_N$. If on the other hand the final specific entropy exceeds the critical entropy, the expansion will lead to a oversaturated (or quenched) nuclear vapor state, with density $0.1 - 0.4n_0$, below the critical nuclear density. The instabilities will lead now to the condensation of a nuclear liquid droplet with an interior nuclear density, $n_1 + n_0 \approx n_N$.

3. Condition for the instabilities to grow

Equations (8,10) determine the condition for κ becoming real, but its sign remains undefined. From equations (5,9) it is easy to see, that the amplitude of the perturbation will depend on the sign of κ: positive κ-s will cause exponentially increasing perturbation, while perturbations corresponding to negative κ will be damped rapidly. To determine the sign of the κ we have to see, how the energy

	β (MeV fm^3)	γ (MeV fm$^{3(\sigma+1)}$)	σ	V_0 (MeV fm)	μ (fm^{-1})
SOFT	1051.76	1107.41	1/6	83.5	2
HARD	365.0	808.65	1	83.5	2

Table 1: The parameterization of the SOFT and HARD EOS

changes due to the perturbation. If the configuration with the perturbation acquires smaller total nuclear energy (the total energy without the flow) than the unperturbed system can we speak about growing instabilities, that is flow can develop to take extra matter into the perturbation.

In the following we consider density dependent Skyrme forces with an averaged Yukawa term. The total nuclear energy of the system can be written as

$$E = E_{kin} - \beta \int d^3r \; n^2(\vec{r}\,) + \gamma \int d^3r \; n^{\sigma+2}(\vec{r}\,) -$$

$$V_0 \int d^3r_1 d^3r_2 \; n(\vec{r}_1)n(\vec{r}_2)\frac{e^{-\mu|\vec{r}_1-\vec{r}_2|}}{|\,\vec{r}_1 - \vec{r}_2\,|}, \tag{11}$$

where the values of β, γ, σ, V_0 and μ are the same as in Ref. [13] and summarized in Table 1. Let us consider a system which is initially homogeneous and has constant density $n_0(\tau) = n_{fr}(\tau_{fr}/\tau)^3$. Introducing now a small perturbation in the density in a way that the total mass number has to be conserved, we obtain a density distribution

$$n(\tau, \vec{r}\,) = n_0(\tau) + n_1(\tau, \vec{r}\,) - \frac{\Gamma}{\Omega} \tag{12}$$

where $\Gamma = \int d^3r \; n_1(\tau, \vec{r}\,)$, Ω is the volume of the system, and n_1 is assumed to be small compared to n_0. If the initial configuration is such that the formation of a perturbation may lead to a decrease of the energy such perturbations will appear and grow spontaneously. This will lead to a multifragmentation of the system. We consider here the energy of the system and not the Helmholtz free energy because we are describing a post freeze-out situation when we do not have a heath bath any more.

Substituting (12) into (11) and expanding in terms of n_1 up to the second order, we evaluate the total energy of the perturbed system. For large enough systems the terms containing Γ^2/Ω can also be neglected. Thus the total nuclear energy can be written as $E = E_0 + \Delta E$, where

$$\frac{E_0}{A} = \frac{E_{\text{kin}}(n_0)}{A} - \left(\beta + \frac{4\pi V_0}{\mu^2}\right)n_0 + \gamma n_0^2 \quad , \tag{13}$$

and $A = \int d^3r \ n(\tau, \vec{r}) = \int d^3r \ n_0 = n_0\Omega$. The change of the energy due to the formation of the perturbation in the second order of n_1 (the first order terms cancel due to the mass number conservation: Eq. 12) is then

$$\Delta E = \Delta E_{\text{kin}} - \left(\beta + \frac{4\pi V_0}{\mu^2} - \frac{1}{2}(\sigma + 1)(\sigma + 2)\gamma n_0^\sigma \right) \overline{N}_1 + \Delta\varepsilon_{\text{surf}}\overline{N}_1,$$

$$\Delta\varepsilon_{\text{surf}} = \left[\frac{4\pi V_0}{\mu^2} - \frac{V_0}{\overline{N}_1} \int d^3r_1 d^3r_2 \ n_1(\vec{r}_1)n_1(\vec{r}_2)\frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\ \vec{r}_1 - \vec{r}_2\ |} \right], \tag{14}$$

where $\overline{N}_1 = \int d^3r \ n_1^2(\vec{r})$. The term $\Delta\varepsilon_{\text{surf}}$ is the surface correction due to the Yukawa interaction. The sign of ΔE will determine whether an instability may increase or will be damped.

It is not immediately clear, whether the kinetic energy gives a contribution to ΔE or not. In thermal systems at high temperature and low density, where the exact value of the Fermi momentum is not too important we can assume that $f = f_0 + f_1$, where $f_1 \approx [n_1(\tau, \vec{r})/n_0] \ f_0$, and the kinetic energy depends only linearly on n_1, that is a density perturbation will not cause a change of total kinetic energy. For a isotherm, degenerate system, where the kinetic energy is nearly proportional to $n^{5/3}$, a perturbation may give a contribution to ΔE. Here we are studying a post freeze-out situation out of thermal equilibrium. Now the kinetic energy depends on the form of our perturbed phase space distribution function, $f_s(\tau, \vec{P})$, we choose or obtain. This may have different characteristics depending on the density of our frozen-out system. Thus we will examine both situations, the one without the kinetic energy contribution (ΔE_1), and the one with (ΔE_2).

The surface correction, $\Delta\varepsilon_{surf}$, for the cases of plane wave and spherical perturbations considered in Section 2 are:

$$\text{Case A} \qquad \frac{4\pi V_0}{\mu^2}\left(1 - \frac{\mu^2}{\mu^2 + k^2} \right)$$
$$\text{Case B} \qquad (4\pi V_0/\mu^2)\left[1 - g(R^*, k) \right]$$

where g is a complicated function of R^* and k (see Appendix C). For the used forces in Eq. (11) in the case A ΔE turns out to be as seen in Ref. [13].

$$\Delta E_{kin} - \left(\beta + \frac{4\pi V_0}{k^2 + \mu^2} - \frac{(\sigma + 1)(\sigma + 2)}{2}\gamma n_0^\sigma \right) \overline{N}_1 < 0.$$

We get a negative change in the energy only for $k < k_{\text{crit}}$, where the critical value of k is

$$k_{\text{crit}}^2 = -\mu^2 + \frac{4\pi V_0}{\frac{\Delta E_{kin}}{\overline{N}_1} - \beta + \frac{(\sigma + 1)(\sigma + 2)}{2}\gamma n_0^\sigma}$$

In case B the required negativity of ΔE can be expressed in a more complicated way.
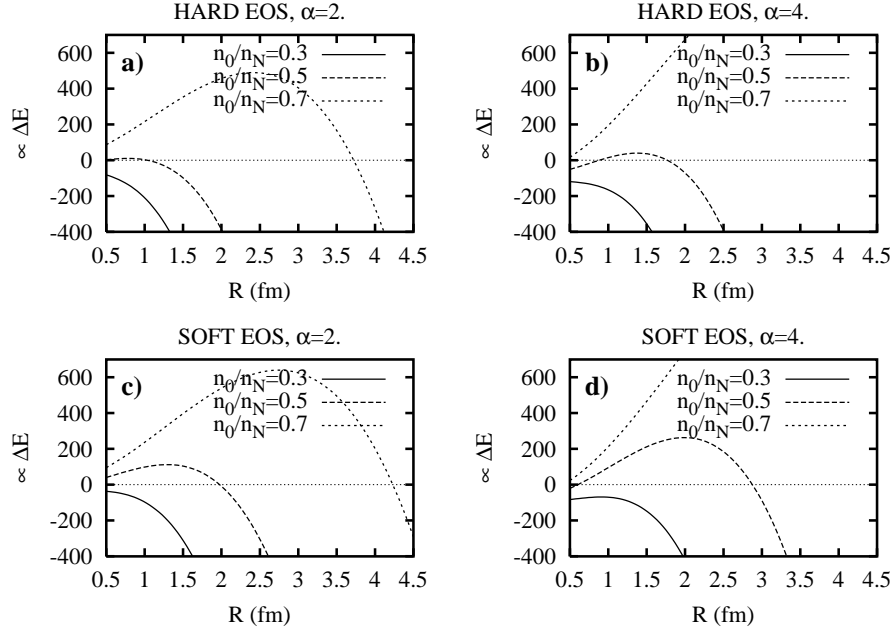
Figure 1: The dependence of the energy difference, ΔE, caused by the formation of a bubble of radius R, without the kinetic term on the radius of the bubble for two different diffuseness coefficients, $\alpha = k\tau_{fr}/\tau$, and two equations of state. The energy difference is evaluated for three post-freeze-out densities $n_0/n_N = 0.3$, 0.4 and 0.7. The maxima of the curves (if any) is at the critical radius, R^*

4. Results

In the following we present here the results for a spherical droplet perturbation considered in Section 2.2. The total energy of the system is given in Eq. (11) and the energy change due to the bubble formation in Eq. (14). The surface energy, $\Delta\varepsilon_{\mathrm{surf}}$, energy given in Eq. (32) depends strongly on $\alpha = k\tau_{fr}/\tau$ and on the radius R^* of the bubble. To see the effect of the bubble shape on the energy change ΔE we give this change for different α and density values as the function of the bubble radius. In Fig. 1 we assumed that the kinetic energy does not give any contribution in second order to the energy, in Fig. 2 we considered a ground state Fermi kinetic energy contribution. As one can see, the effect of the surface term is larger if we have sharper surfaces (larger α values). As one expects, the effect of the surface energy, $\Delta\varepsilon_{\mathrm{surf}}$, decreases for large radii. As a first step we considered the solution of Eq. (10) for $\kappa=0$, and compared the condition of the dynamical instabilities to grow with that of the mechanical instability for infinite systems. Without surface term the two condition are the same, as was pointed out already in Ref. [12]. With the surface term the instability region decreases, just as in Ref. [13]. In Fig. 3 we give the $n - T$ curve of instability region for different α and R^* values. With small supercooling first big droplets can nucleate and grow, then with stronger supercooling smaller droplets can also be formed. For big droplets ($R^* = 4.5$ fm) the effect of the surface term is almost negligible.
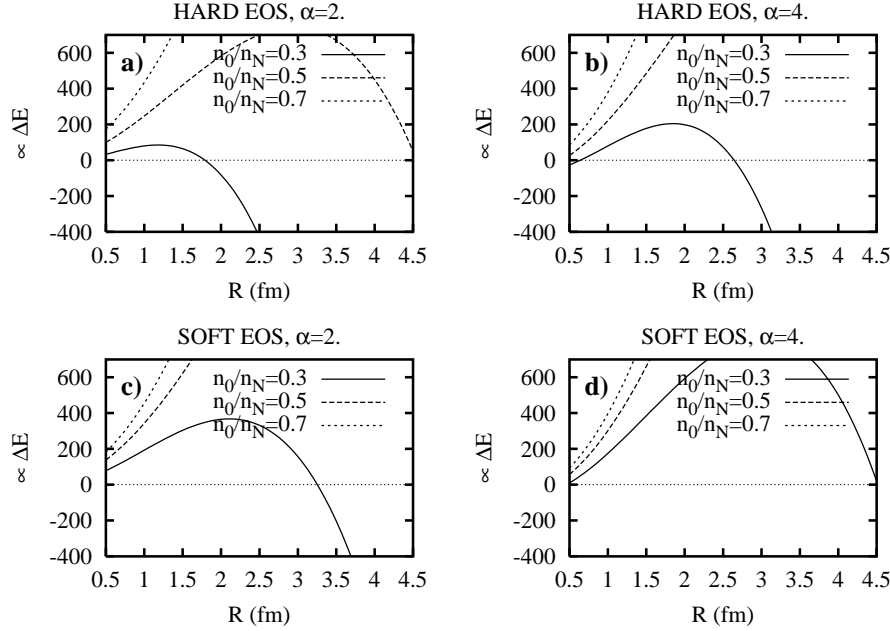
Figure 2: The same as Fig. 1 with kinetic energy term included

If we wait longer after the freeze-out in the expanding system, i.e., we increase τ and thus having a smaller α, ($\alpha = \kappa \tau_{fr}/\tau$) we have the possibility of instabilities earlier, with smaller supercooling. The calculations are done both for soft and hard equation of state. One sees, that the effect of the surface term is more significant for the soft equation of state.

As the next step we want to determine the break up of the system starting from different freeze out densities times and temperatures. A reasonable freeze out density should be in the range of $n_0 = 0.08 - 0.14$ fm^{-3}. In our calculation we choose $n_{fr} = 0.1$ fm^{-3}. Other freeze out densities can be considered simply rescaling τ_{fr}, μ_{fr} and T_{fr} to keep the relation $(n_0/n_{fr})^{2/3} = (T/T_{fr}) = (\tau_{fr}/\tau)^2$ and μ_{fr}/T_{fr} constant (see the remarks after Eq. (3)). The freeze out time τ_{fr} defines the flow energy of the system as $E_{\text{flow}}/A = \dfrac{3}{5}\dfrac{R^2}{\tau_{fr}^2}$ for a system with radius R. We assume, that the total excitation energy of the system is large enough to reach the break up densities [15], so if that condition is fulfilled, the freeze out time is not defined in the model, and the time scale is not fixed. We followed the paths along the trajectories in the $n - T$ plane from different initial freeze out configurations. The instability condition Eq. (10) is examined along the trajectories, and the time of the solution for growing instability (break up) can be expressed as $\Delta\tau = \tau - \tau_{fr} = \tau_{fr}\left[\left(\dfrac{n_{fr}}{n}\right)^{1/3} - 1\right]$ after the freeze out.

In Fig. 4 we show the part of the trajectories of the post-freeze-out expansion denoted by dotted line where the instability condition is fulfilled and the energy change, ΔE_1, is negative. We found, that whenever condition (10) is fulfilled, this
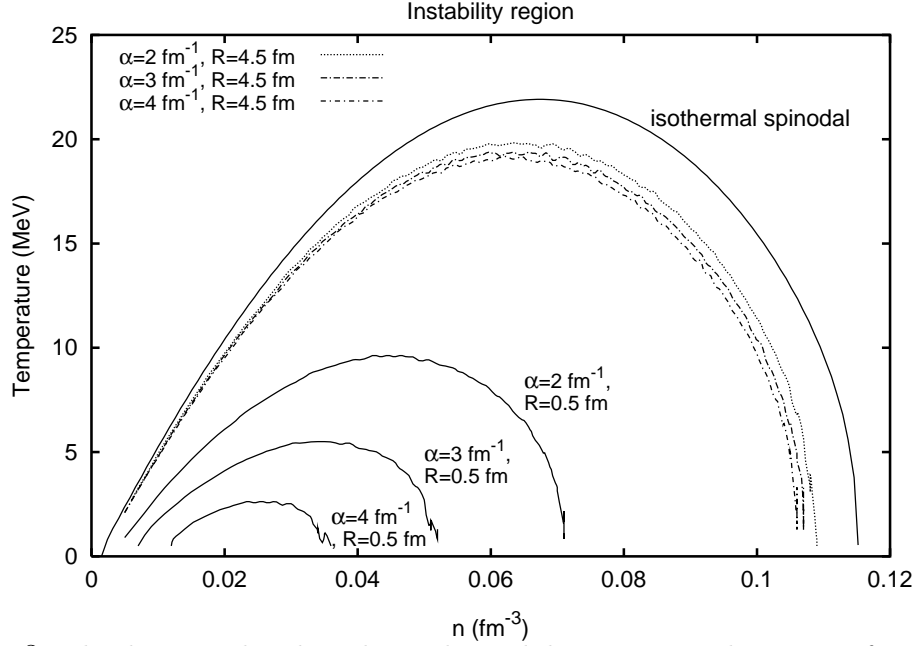
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Figure 3: The dynamical and mechanical instability regions. The region of mechanical instability is bordered by the isothermal spinodal, where the isotherm sound speed vanishes and becomes imaginary. For large size (radius R) of the critical bubbles or for systems without surface energy the dynamical instability region is the same as the mechanical one. The break up instability depends on the radius R and on the stretching of the system parameterized by α

energy change is always negative. The more strict condition, using ΔE_2, however, excludes some part of the trajectories (solid line) at temperatures above 10 MeV for the hard equation of state. For the soft equation of state there is no such exclusion, nevertheless, the instability region is smaller. We give the results for the two equations of state and different parameters for both. For comparison we show the boundary of the isothermal mechanical instability (which corresponds to the $R^* \to \infty$ situation). In the parameter region we count as physical ($R^* \approx$ 2–3 fm, $k \approx 4 - 6$ fm^{-1}) there are no significant changes on this parameters.

Following the trajectory of the post-freeze-out expansion starting from a given initial n_{fr}, T_{fr} configuration the instability condition Eq. (10) has solutions for different S or κ values. For high densities and temperatures S is negative, that is there are no real solution for κ. As the density (and correspondingly the temperature) decreases it continuously becomes positive. The speed of growth of the instabilities is determined by $\kappa = \dfrac{\tau_{fr}}{\tau} \dfrac{\tau_{th}}{\tau} \sqrt{\dfrac{2T_{fr}S}{m(R^*)^2}}$. Evaluating this expression we assumed that $\tau_{th}/\tau \approx 1$. In Fig. 5 κ is shown along given expansion trajectories ($k=4$ fm^{-1} $R= 2$ fm, $T_{fr} = 1$ MeV, 16 MeV and 21 MeV) for the hard equation of state. The speed κ, the instabilities are developing with, is changing, first it increases as the nucleus evolves to smaller densities, later it decreases back to zero. The time scale of the expansion for the radius of the perturbations from
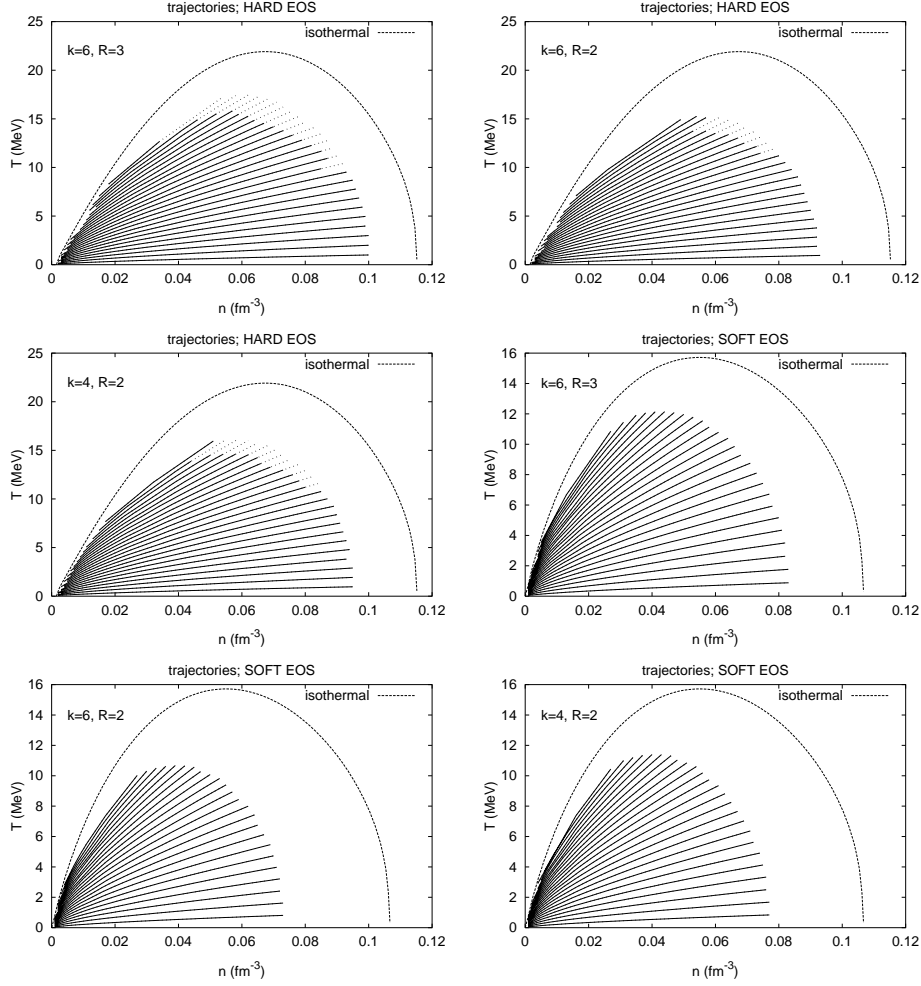
Figure 4: The trajectories of an adiabatic post-freeze-out expansion starting at the same, $n_{fr} = 0.1$ fm^{-3} freeze out density. The different lines are originated from different freeze out temperatures. The solid lines correspond to the case where the energy difference with the kinetic term is negative, the dotted line is the case where the energy difference without the kinetic term is negative. The latter ones define the wider region. The possibility of dynamical post-freeze-out instability opens only after some penetration into the domain of the isothermal spinodal, while thermally dominated homogeneous nucleation may start immediately, although slowly
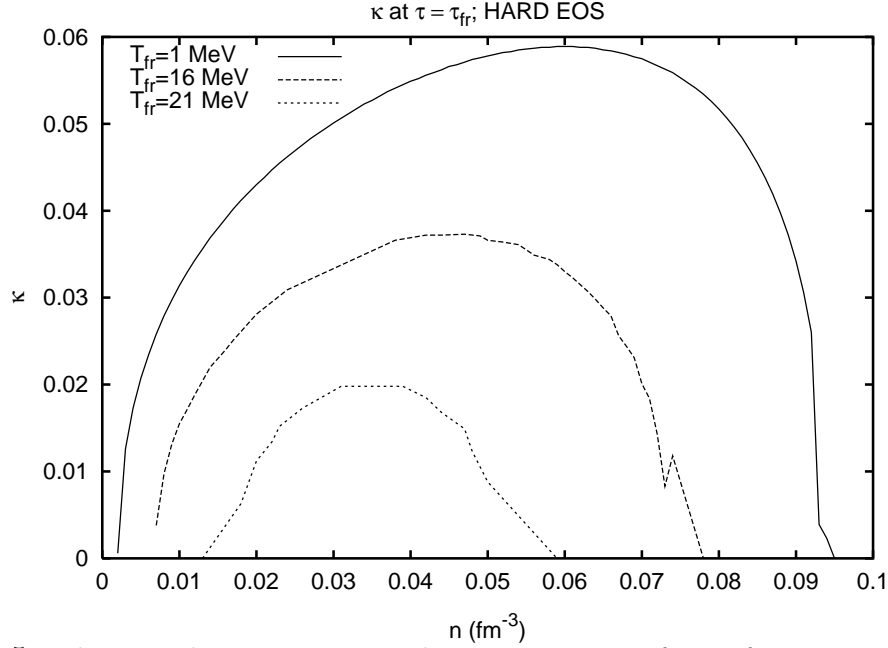
Figure 5: The growth parameter, κ, along trajectories of post-freeze-out expansion starting at $n_{fr} = 0.1$ fm^{-3} and at the given freeze-out temperatures

Fig. 5 gives ≈ 20 fm/c for the $\kappa \approx 0.06$. However, the growth rate of the instability from Eq. (9) is a double exponential: $n_c \, exp\left(k\tau_{fr}/\tau_{th} \, R^* e^{\kappa\Delta(\tau-\tau_{th})}\right)$, which is much faster. This region of the exponentially developing instabilities breaks up the system.

Appendix A: Dispersion relation for plane wave perturbation

Let us consider a plane wave perturbation in another form than in Ref. [12] emphasizing our time scale

$$f_1(\tau,\vec{r},\vec{p}) = f_s(\tau,\vec{P}) \, \exp\left[i\frac{\vec{k}\tau_{fr}}{\tau}\vec{r} + \kappa(\tau - \tau_{th})\right], \qquad (15)$$

where it is taken into account, that the wave number, $\vec{k}$, scales with τ, as all other parameters of the flow. Thus k is a constant, independent of τ. Here f_1 is a phase space distribution which should satisfy the Vlasov equation (2), and it represents a plane wave with growing amplitude if κ is a real positive number. Since we assume small perturbations only, this solution is valid only for a short time after the instability starts to grow. (Frequently in similar studies the perturbation is studied in a form containing an $\exp(i\omega\tau)$ term, and the opening of the channel of instability is indicated when ω becomes imaginary. Thus perturbations preceding

τ_{th} will not grow. This approach is basically equivalent to the one presented here, however, we wanted to emphasize that the instability may grow only after τ_{th}.)

As it was already mentioned we also assume that $f_s(\tau, \vec{P})$ has a characteristic time dependence on the slow scale (i) of the post freeze-out expansion, so that its time-derivatives are negligible compared to other time-derivatives of $f_1(\tau, \vec{r}, \vec{p})$ corresponding to rapid inequilibrium processes (ii) like $\exp[\kappa(\tau - \tau_{th})]$. Furthermore, we assume that f_s depends on the LR momentum, $\vec{P}$ only, i.e., it does not have any other dependence on $\vec{r}$ other than what is included in $\vec{P}$.

We search for such solutions of the Vlasov equation, $f_1(\tau, \vec{r}, \vec{p})$ and $f_s(\tau, \vec{P})$. We assume that such solutions can be obtained only some time, τ, after the freeze-out at τ_{fr}, i.e. at $\tau_{th} > \tau_{fr}$.

Using the above form, (5), of perturbation the density change arising from this perturbation is:

$$n_1(\tau, \vec{r}) = n_s(\tau) \; \exp[i\frac{\vec{k}\tau_{fr}}{\tau}\vec{r} + \kappa(\tau - \tau_{th})].$$

Inserting a plane wave perturbation Eq. (5) into the Vlasov equation we obtain for the force used in Eq. (11)

$$f_s(\tau, \vec{P}) \; \left[\kappa + \frac{i\vec{k}\tau_{fr}}{m\tau}(\vec{p} - m\vec{r}/\tau)\right] -$$

$$\frac{\partial U(k)}{\partial n} \; n_s(\tau)\frac{i\vec{k}\tau_{fr}}{\tau} \; \left(-\frac{f_0^2}{C}\right)\frac{[\vec{p} - m\vec{r}/\tau]}{mT_{eff}}\exp\left[\frac{[\vec{p} - m\vec{r}/\tau]^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right] = 0 \quad, \quad (16)$$

where
$$\frac{\partial U(k)}{\partial n} = -2\beta + \gamma(\sigma + 1)(\sigma + 2) \; n_0^\sigma - \frac{4\pi V_0}{\mu^2 + k^2}$$

as in Ref. [13].

Note that the momentum appears only inside expressions of the LR momentum, $\vec{p} - m\vec{r}/\tau$, thus we can integrate it out in the LR frame. Thus from (16) we can express f_s, and integrating it over the LR momentum, $\vec{P} = \vec{p} - m\vec{r}/\tau$, we obtain $n_s(\tau)$, which then can be eliminated from both sides yielding:

$$1 = -\frac{1}{T_{eff}C} \; \frac{\partial U(k)}{\partial n} \; \int d^3P f_0^2(\vec{P})\frac{i\tau_{fr}\vec{k}\vec{P}/\tau}{m\kappa + \frac{i\vec{k}\tau_{fr}}{\tau}\vec{P}}\exp\left[\frac{\vec{P}^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right] \quad (17)$$

We can separate the variable of the integral to a parallel, $P_{||}$, and an orthogonal, $\vec{P}_\perp$, component with respect to $\vec{k}$, and the integration over the components perpendicular to k can be performed. This yields

$$1 = -2\pi mC \; \frac{\partial U(k)}{\partial n} \int\limits_{-\infty}^{\infty} dP_{||}\frac{ikP_{||}\tau_{fr}/\tau}{m\kappa + ikP_{||}\tau_{fr}/\tau}\left\{1 + \exp\left[\frac{P_{||}^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right]\right\}^{-1}. \quad (18)$$

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Only symmetric functions contribute to this integral, so we symmetrize it by multiplying both the numerator and denominator by $m\kappa - ikP_\parallel\tau_{fr}/\tau$, and then dropping the antisymmetric term we end up having

$$1 = -4\pi mC \, \frac{\partial U}{\partial n} \int\limits_0^\infty dP_\parallel \frac{P_\parallel^2}{[m\kappa\tau/(k\tau_{fr})]^2 + P_\parallel^2} \left\{1 + \exp\left[\frac{P_\parallel^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right]\right\}^{-1}. \quad (19)$$

Introducing a new variable, $y = P_\parallel^2/(2mT_{eff})$, a straightforward calculation will lead to the dispersion relation in the integral form:

$$1 = -2\pi mC\sqrt{2mT_{eff}} \, \frac{\partial U}{\partial n} \int\limits_0^\infty \frac{dy \, \sqrt{y}}{(y+s)[1 + e^{y - \mu_{fr}/T_{fr}}]}, \quad (20)$$

where $s = m\kappa^2\tau^2/(2k^2\tau_{fr}^2 T_{eff}) = m\kappa^2\tau^4/(2k^2 T_{fr}\tau_{fr}^4)$.

Appendix B: Dispersion relation for spherical perturbation

For the sake of simplicity let us first study a spherical cusp perturbation centered around some interior point $\vec{r}_c(\tau) = \vec{r}_0\tau/\tau_{fr}$ of the type

$$f_1 = \exp\left[-k\tau_{fr}|\vec{r} - \vec{r}_0\tau/\tau_{fr}|/\tau + \kappa(\tau - \tau_{th})\right] f_s, \quad (21)$$

where the center of the perturbation moves along the scaling expansion, and this center was at point $\vec{r}_0$ at the time of the freeze-out. As we discussed it in the introduction we can assume that $\vec{r}_0 = 0$, so without loosing the generality of the assumption, $|\vec{r} - \vec{r}_0| \longrightarrow |\vec{r}| = r$, since the interior points of the expanding nuclear system are equivalent. Thus

$$f_1 = \exp\left[-k\tau_{fr}r/\tau + \kappa(\tau - \tau_{th})\right] f_s, \quad (22)$$

can serve to study the perturbation just as well. Although this functional form of perturbation has a singularity in the center this will not be essential for our study, and it could be removed by assuming more complicated functional forms for the perturbation which would not show such a singularity.

However, including the center of the perturbation, $\vec{r}_c$, explicitly will allow us later to discuss interactions (e.g. fusion, repulsion, etc.) of two (or more) elementary perturbations. Thus we will follow this somewhat more complicated derivation although it is not necessary at the moment.

We will not explicitly define the form of f_s at this stage, unlike in the case of plane wave perturbations. Instead we will consider the integrals of f_1 and f_s. First the norms:

$$n_1 = \int d^3p \, f_1 = \exp\left[-k\frac{\tau_{fr}}{\tau}\left|\vec{r} - \vec{r}_0\frac{\tau}{\tau_{fr}}\right| + \kappa(\tau - \tau_{th})\right] n_s,$$

and $n_s = \int d^3p f_s$, where we require that $n_s = n_s(\tau)$ does not depend on $\vec{r}$. Second the projections orthogonal to $(\vec{r} - \vec{r}_c)$, i.e.,

$$g_1(\tau, \vec{r}, P_{||}) = 2\pi \int\limits_0^\infty P_\perp dP_\perp f_1 = \exp\left[-k\frac{\tau_{fr}}{\tau}\left|\vec{r} - \vec{r}_0\frac{\tau}{\tau_{fr}}\right| + \kappa(\tau - \tau_{th})\right] g_s.$$

Here $g_s(\tau, P_{||}) = 2\pi \int P_\perp dP_\perp f_s$, where we require that $g_s = g_s(\tau, P_{||})$ does not depend on $\vec{r}$. We chose our coordinate system so that $P_{||}$ is parallel to $(\vec{r} - \vec{r}_c)$, and $\vec{P}_\perp$ is orthogonal to it. These constraints are the required implicit constraints on the choice of f_s.

Inserting Eq. (21) into the Vlasov equation and integrating it over $d^2 P_\perp$ we obtain

$$g_s\left[\kappa + \frac{k\tau_{fr}}{m\tau}\vec{p}\frac{\vec{r} - \vec{r}_0\tau/\tau_{fr}}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|} - \frac{k\tau_{fr}}{\tau^2}|\vec{r} - \vec{r}_0\tau/\tau_{fr}| - \frac{k\vec{r}_0}{\tau}\frac{\vec{r} - \vec{r}_0\tau/\tau_{fr}}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}\right] -$$

$$2\pi C n_s\frac{\partial U}{\partial n}\frac{k\tau_{fr}}{\tau}\frac{\vec{r} - \vec{r}_0\tau/\tau_{fr}}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}P_{||}\left\{1 + \exp\left[\frac{P_{||}^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right]\right\}^{-1} = 0, \qquad (23)$$

where for an averaged Yukawa surface term U is only the function of n.

Performing products in the first term and taking into account that $\vec{P}\frac{\vec{r} - \vec{r}_0\tau/\tau_{fr}}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}$ $= P_{||}$ we obtain

$$g_s\left[\kappa + \frac{k\tau_{fr}}{m\tau}[\vec{p} - m\vec{r}/\tau]\frac{\vec{r} - \vec{r}_0\tau/\tau_{fr}}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}\right] =$$

$$g_s\left[\kappa + \frac{k\tau_{fr}}{m\tau}P_{||}\frac{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}{|\vec{r} - \vec{r}_0\tau/\tau_{fr}|}\right] = g_s\left[\kappa + \frac{k\tau_{fr}}{m\tau}P_{||}\right],$$

so that Eq. (23) takes the form

$$g_s\left[\kappa + \frac{k\tau_{fr}}{m\tau}P_{||}\right] - 2\pi C n_s\frac{\partial U}{\partial n}\frac{k\tau_{fr}}{\tau}P_{||}\left\{1 + \exp\left[\frac{P_{||}^2}{2mT_{eff}} - \frac{\mu_{eff}}{T_{eff}}\right]\right\}^{-1} = 0. \quad (24)$$

We see that the position of the center of the perturbation, $\vec{r}_0$, has dropped out. We indicated this symmetry already in the introduction when we pointed out that in the spherical scaling expansion all interior points are equivalent in the sense of their LR features. We would obviously get the same result assuming $\vec{r}_0 \equiv 0$ from Eq. (21) on in the course of this derivation.

Dividing both sides by $(\kappa + \frac{k\tau_{fr}}{m\tau}P_{||})$ and integrating over $dP_{||}$ leads to

$$1 = 2\pi C\frac{\partial U}{\partial n}\frac{k\tau_{fr}}{\tau}\int\limits_{-\infty}^\infty P_{||}dP_{||}\left[\kappa + \frac{k\tau_{fr}}{m\tau}P_{||}\right]^{-1}\left\{1 + \exp\left[\frac{P_{||}^2}{2mT_{eff}} - \frac{\mu_{fr}}{T_{fr}}\right]\right\}^{-1}. \quad (25)$$

Multiplying both the denominator and the numerator by $\kappa - \frac{k\tau_{fr}}{m\tau}P_{\parallel}$ and then dropping the antisymmetric part, which does not contribute to the integral we obtain

$$1 = -4\pi mC\frac{\partial U}{\partial n}\int_0^\infty dP_{\parallel}P_{\parallel}^2\left[\frac{m^2\tau^2\kappa^2}{k^2\tau_{fr}^2} - P_{\parallel}^2\right]^{-1}\left\{1 + \exp\left[\frac{P_{\parallel}^2}{2mT_{eff}} - \frac{\mu_{fr}}{T_{fr}}\right]\right\}^{-1}. \quad (26)$$

The same way as we got the dispersion relation in the case of plane wave perturbation from Eq. (19) we get now the relation

$$1 = -2\pi mC\sqrt{2mT_{eff}}\,\frac{\partial U}{\partial n}\int_0^\infty \frac{dy\,\sqrt{y}}{(y-s)[1 + e^{y-\mu_{fr}/T_{fr}}]}, \quad (27)$$

where $s = m\kappa^2\tau^2/(2k^2\tau_{fr}^2 T_{eff}) = m\kappa^2\tau^4/(2k^2 T_{fr}\tau_{fr}^4)$.

If we have a spherical droplet perturbation of a finite radius described by Eq. (9) instead of a cusp, Eq. (21), the dispersion relation can be obtained from (27) by making the transformation $\kappa \longrightarrow \kappa k R^* \tau_{fr}/\tau_{th}$ arising from comparing the form of the two perturbations (21) and (9). This leads to exactly the same equation as the equation above (27) for the spherical cusp perturbations, except that in place of s we have $S = m\kappa^2(R^*)^2\tau^4/(2T_{fr}\tau_{fr}^2\tau_{th}^2)$ in the expression.

Appendix C: Spherical droplet with Yukawa forces

For the profile (9) the Yukawa force can be written as follows

$$V_{\text{Yuk}}(r) = \frac{4\pi V_0}{\mu^2}\left[\frac{\mu}{r}e^{-\mu R}\sinh(\mu r)\left(\frac{R}{\mu} - \frac{R}{\mu + \alpha} + \frac{1}{\mu^2} - \frac{1}{(\mu + \alpha)^2}\right) - 1\right] \quad (28)$$

for $r < R(\tau)$, where $\alpha = k\tau_{fr}/\tau$, and

$$\begin{aligned}
V_{\text{Yuk}}(r) &= -4\pi V_0 n_c e^{-\alpha(r-R)}\left[-\frac{1}{\alpha^2 - \mu^2} - \frac{2\alpha}{r(\alpha^2 - \mu^2)^2}\right. \\
&+ \frac{e^{(\alpha-\mu)(r-R)}}{2r\mu}\left(\frac{R}{\mu} - \frac{1}{\mu^2} + \frac{R}{\alpha - \mu} + \frac{1}{(\alpha - \mu)^2}\right) \\
&+ \left.\frac{e^{(\alpha-\mu)r}}{2r\mu e^{(\alpha+\mu)R}}\left(\frac{R}{\mu} + \frac{1}{\mu^2} - \frac{R}{\alpha + \mu} - \frac{1}{(\alpha + \mu)^2}\right)\right]
\end{aligned} \quad (29)$$

for $r > R(\tau)$.

The Yukawa energy can be written as

$$\begin{aligned}
E_{\text{Yuk}} &= \int d^3r\, V_{\text{Yuk}}(\vec{r})n_1(\vec{r}) \\
&= -\frac{4\pi V_0}{\mu^2}n_c^2\frac{4\pi}{3}\left[R^3 - \frac{3}{2}R^2\frac{\alpha^2 - \alpha\mu - \mu^2}{\mu\alpha(\mu + \alpha)}\right.
\end{aligned}$$

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$$+ \ \frac{3}{2}R\left(-\frac{2}{\mu}\frac{1}{(\mu+\alpha)} + \frac{1}{(\mu+\alpha)^2} + \frac{1}{\alpha^2}\right)$$

$$+ \ \frac{3}{2}\ \left(\frac{(2\alpha+\mu)\mu}{2\alpha^3(\mu+\alpha)^2} - \frac{2}{\mu(\mu+\alpha)^2} + \frac{1}{\mu^3}\right) \tag{30}$$

$$- \ \frac{3}{2}\mu e^{-2\mu R}\left(\frac{R\alpha}{\mu(\mu+\alpha)} - \frac{1}{(\mu+\alpha)^2} + \frac{1}{\mu^2}\right)^2\Bigg]$$

One sees that substituting expression (28) and (30) into the Vlasov equation, the perturbation (10) is not a solution of it. However, for sharply decreasing surfaces $\alpha R > 1$ and we can consider instead of V_{Yuk} the average of it. That is, we use a surface term, which is given as the average of the Yukawa interaction. Instead of (30) we introduce for $r > R(\tau)$ (see Eq. (11))

$$U_{\text{surf}} \ = \ U^0_{\text{surf}}n_1(\vec{r}) \ = \ \frac{4\pi V_0}{\mu^2}\left[1 - \overline{V}_{\text{Yuk}}\right] = \tag{31}$$

$$= \ \frac{4\pi V_0}{\mu^2}\ n_1(\vec{r})\ \frac{\alpha}{\mu(\mu+\alpha)}\ \frac{R + \frac{2\alpha+\mu}{2\alpha(\alpha+\mu)}}{R^2 + \frac{R}{\alpha} + \frac{1}{2\alpha^2}}$$

The surface energy term in Eq. (11) can be written using (30) and neglecting the terms proportional to $e^{-2\mu R}$ (these terms are small) as

$$\Delta\varepsilon_{\text{surf}} \ = \ \frac{4\pi V_0}{\mu^2}\ - \ \frac{E_{\text{Yuk}}}{\overline{N}_1}$$

$$= \ \frac{1}{\overline{N}_1}\frac{4\pi V_0}{\mu^2}\left[\frac{R^2\alpha}{2\mu(\mu+\alpha)} + \frac{R}{(\mu+\alpha)^2}\frac{2\alpha+\mu}{2\mu} + \frac{4\alpha+\mu}{4\alpha(\mu+\alpha)^2} - \frac{1}{2\mu^3}\right]$$

$$\to \ \frac{6\pi V_0}{\mu^3 R}\ \frac{\alpha}{\mu+\alpha}\ \ \text{for}\ \ \alpha R \gg 1\ . \tag{32}$$

Appendix D: The mechanical instability region

The total energy density of the system can be written as $e_{pot}(n) + e_F$, with the potential energy density $e_{pot}(n)$ and the kinetic (Fermi) energy density e_F. The latter can be written as $e_F = const.\ T^{5/2}F_{3/2}(\mu/T)$, using the integrals $F_{i/2}(\eta) = \int_0^\infty dx\frac{x^{i/2}}{1+\exp(x-\eta)}$. The density can be expressed as $n = const.\ T^{3/2}F_{1/2}(\mu/T)$. Isotherm expansion, $dT = 0$, leads to the change of Fermi energy

$$de_F = 3T\frac{F_{1/2}(\mu/T)}{F_{-1/2}(\mu/T)}dn$$

The pressure should be calculated from the free energy density $f(T, n) = e - Ts$, with the entropy density $s = \frac{5}{3}e_F/T - n\mu/T$:

$$p = n^2\frac{\partial f/n}{\partial n} = n\frac{\partial e_{pot}(n)}{\partial n} - e_{pot}(n) + \frac{2}{3}e_F$$

The region of mechanical instability where the derivative of the pressure above with respect to the density at constant temperature is negative:

$$\frac{dp}{dn} = n\frac{\partial^2 e_{pot}(n)}{\partial n^2} + 2T\frac{F_{1/2}(\mu/T)}{F_{-1/2}(\mu/T)} = n\frac{\partial U}{\partial n} + 2T\frac{F_{1/2}(\mu/T)}{F_{-1/2}(\mu/T)}$$

The onset of the instability is determined then by

$$1 = -\frac{1}{2}cT^{1/2}\frac{\partial U}{\partial n}F_{-1/2}(\mu/T) \quad , \quad c = \frac{g}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \tag{33}$$

The condition of the dynamical instability derived in Appendix B is

$$1 = -\frac{2\pi\tau_{fr}m}{\tau}\frac{g}{(2\pi\hbar)^3}\sqrt{2mT_{fr}}\frac{\partial U}{\partial n}\int_0^\infty \frac{dy\sqrt{y}}{(y-s)\left(1+\exp(y-\mu_{eff}/T_{eff})\right)}$$

substituting $\tau_{fr}/\tau = (T/T_{fr})^{1/2}$ for the critical mode ($s = 0$ growing) one gets

$$1 = -\frac{1}{2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{g}{4\pi^2}T^{1/2}\frac{\partial U}{\partial n}F_{-1/2}(\mu_{eff}/T_{eff}) \quad , \tag{34}$$

which is the same as the condition for the isothermal spinodal.

Appendix E: Critical radius

Let us introduce the notation $\langle n_1^2\rangle \frac{4\pi}{3} R^3 \equiv \overline{N}_1$ and rewrite Eq. (32) as

$$\Delta E_{\mathrm{surf}} = \frac{8\pi^2 V_0}{\mu^2}\frac{\alpha\langle n_1^2\rangle}{R\mu(\mu+\alpha)}R^2 \equiv a(\tau)R^2. \tag{35}$$

Using Eq. (14) the total energy change due to the formation of a droplet of size R can be cast in the form

$$\Delta E(R) = \Delta E_{\mathrm{kin}} - \underbrace{\left(\frac{\beta}{3} + \frac{4\pi V_0}{3\mu^2} - \frac{(\sigma+1)(\sigma+2)}{2}\gamma n_0^\sigma\right)4\pi\langle n_1^2\rangle}_{\equiv b}R^3 + aR^2.$$

Thus, assuming that the contribution of kinetic energy is independent of R we obtain the critical radius from $\partial\Delta E/\partial R = 0$

$$R_{\mathrm{crit}}^* = \frac{2a(\tau)}{3b}.$$

At R_{crit}^* the function $\Delta E(R)$ has its maximum, thus bubbles or droplets smaller than R_{crit}^* shrink while larger than R_{crit}^* grow.

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Notes

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